Stretto di Messina

Concessionaria per la progettazione, realizzazione e gestione del collegamento stabile tra la Sicilia e il Continente Organismo di Diritto Pubblico
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EurolinK

PONTE SULLO STRETTO DI MESSINA

PROGETTO DEFINITIVO

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IL PROGETTISTA	IL CONTRAENTE GENERALE	STRETTO DI MESSINA	STRETTO DI MESSINA
COWI Ing. E.M. Veje Dott. Ing. E. Pagani Ordine Ingegneri Milano n° 15408 EurolinK	Project Manager (Ing. P.P. Marcheselli)	Direttore Generale e RUP Validazione (Ing. G. Fiammenghi)	Amministratore Delegato (Dott. P. Ciucci)

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1 Executive Summary

For an accurate modelling of the bridge structural response, the deformability of the soil-foundation system must be taken into account. Specifically, at each of the contact points of the structural model with the soil, this deformability is expressed through a stiffness matrix that relates the generalised displacement of the contact point to the corresponding generalised force.

The contact points considered in the present study are the tower foundations (two points for each tower), the anchor blocks, and the foundations of the terminal structures.

The stiffness coefficients are evaluated using solutions developed for rigid foundations partly or totally embedded in an elastic continuum of finite depth.

In a first stage, the mechanical properties of the elastic continuum are evaluated by matching the displacements provided by the elastic solutions with those obtained by preliminary non-linear finite-element calculations, for a limited number of degree of freedom of each contact point. Subsequently, the same elastic properties are used to evaluate the stiffness coefficients relative to the remaining degrees of freedom.

The stiffness matrix at each contact point is not diagonal, as the foundation embedment is responsible of some coupling between different degrees of freedom.

2 Foreword

This report illustrates the evaluation of the elastic stiffness coefficients that represent the overall behaviour of the soil-foundation system for the contact points of the bridge with the ground surface.

A specific matrix coefficient is associated to each contact point, that contains the stiffness coefficients relating forces and moments to the corresponding displacements and rotations.

The contact points considered in the present study are the tower foundations (two points for each tower), the anchor blocks, and the foundations of the terminal viaducts.

The stiffness coefficients were evaluated using solutions developed for rigid foundations embedded in an elastic layer of finite depth. The elastic properties of the elastic layer were calibrated, where possible, by matching the displacements provided by the elastic solutions with those obtained in the three-dimensional non-linear finite-element static analyses.

This stiffness coefficients, calibrated on the results of static numerical analyses, represent strictly the foundation behaviour under static loading. For seismic loading, they are to be regarded as a lower bound of the actual foundation stiffness mobilised during an earthquake. This is further discussed in the report CG1003-P-CL-D-P-CG-S4-00-00-00-02_A “Equivalent Stiffness and Damping Matrices for the Soil-Foundation System”, where damping coefficients corresponding to this lower bound stiffness coefficients are also evaluated.

3 Evaluation of Stiffness Coefficients

Gazetas (1991) provides expressions for the dynamic impedances of a foundation embedded in an elastic layer. The static stiffness can be evaluated using these solutions, setting the dynamic amplification factors to one and the damping coefficients to zero.

Solutions for an embedded foundation in an elastic layer are given only for circular shapes and for strip foundations. The tower foundations have a circular shape; for the remaining foundation elements, that have different shapes, the following steps for the computation of the elastic stiffness coefficients were followed:

a. approximate the actual foundation with a circumscribed rectangle;

b. evaluate the stiffness coefficient $k_{(shallow\text{-}r)}$ for the equivalent rectangular foundation resting on the surface of an elastic half-space;

c. evaluate the radius R_{eq} of an equivalent circular foundation resting on the surface of an elastic half-space;

d. calculate the stiffness coefficient $k_{(shallow\text{-}c)}$ of the equivalent circular foundation resting on the surface of an elastic layer;

e. modify the above stiffness coefficient to account for the embedment of the foundation, to evaluate the final stiffness coefficient $k_{(emb\text{-}c)}$;

f. for comparison, modify the stiffness coefficient evaluated in (a) to account for the embedment and evaluate the stiffness coefficient $k_{(emb\text{-}r)}$ for a rectangular foundation embedded in an elastic half-space.

The above calculations must be performed for each of the six degrees of freedom of a rigid foundation. Note that the value of equivalent radius evaluated in (c) depends on the degree of freedom that is being considered.

The analytical expressions used for the six degrees of freedom are reported in Appendix A. They use a frame of reference (*x*, *y*, *z*) in which the *z* axis is vertical and oriented downwards; the *y* axis is horizontal, parallel to the longitudinal direction of the bridge and oriented southward (from Sicily to Calabria); the *x* axis is horizontal and is oriented eastward. Rotations and moments are denoted by the rotation axis; note that clockwise rotations and moments around the *x* axis (*rx*) are positive (see Fig. 1).

The stiffness matrix was first evaluated at the bottom of the equivalent foundation, along its centre-line (points denoted A), and then was transformed to provide the force-displacement relationship for the reference points (denoted G) located by the structural analyst (Cowi Consultants).

Figure 2 shows a layout of an embedded rectangular foundation, with symbols used to denote the geometry. Figures 3 and 4 show the idealised foundations for the Sicily and the Calabria towers. The equivalent foundation includes the circular footings and the volume of soil treated with jet-grouting below the footing, while it neglects the lateral soil treated with a sparser jet-grouting. In Figs 3 and 4, open circles indicate points (A) where the stiffness matrix was originally evaluated, while full circles designate points (G), that is, the position of the elastic elements that represent the soil-foundation stiffness. These reference points correspond with the intersection of the tower axes with the bottom of the circular footings.

Figures 5 and 6 display the schemes used for the anchor blocks: they were regarded as rectangular foundations embedded in an elastic layer. For the anchor blocks, the reference point (G) is the centre of gravity.

Figure 7 and 8 show the idealised foundations of the terminal viaducts on the Sicily and the Calabria shores. Each viaduct has two independent foundations that were given two identical stiffness matrixes. The stiffness matrix of each of these foundations has a structure similar to that of the tower foundations.

For all the foundation elements, coupling occurs between the rotations *rx*, *ry* and the horizontal forces along *x* and *y* (and, equivalently, between the displacements along *x* and y and the moments *ry* and *rx*). These coupled stiffness terms are indicated as $k_{y\text{-}rx}$ and $k_{x\text{-}ry}$.

The calibration of the stiffness coefficients was carried out using as a reference the results of preliminary finite element calculations of the static behaviour of the tower foundations and the anchor blocks.

For the tower foundations, full three-dimensional finite element analyses were carried out, in which the foundation was loaded vertically and in the longitudinal plane. In these numerical analyses the mechanical behaviour of the soil was modelled with an elastic-plastic constitutive model with double isotropic hardening and a Mohr-Coulomb failure criteria. The loads were applied in steps to capture the non-linear behaviour of the soil-foundation system.

The expressions given in appendix A, based on the theory of elasticity, were used to compute the stiffness matrix that reproduced the relationship between the displacements/rotations *y*, *z*, *rx* and the corresponding forces/moment computed with the finite element method. This optimisation was carried out varying the shear modulus *G* (the Poisson coefficient was set to 0.2.) and the coupling stiffness terms $k_{y\text{-}rx}$ and $k_{x\text{-}ry}$ until a good agreement was obtained between the computed and the finite element set of forces or, equivalently, between the finite element displacements and the displacements computed by inverting the stiffness matrix. For the tower foundations, the attention was focused primarily in matching of the forces and displacements in the *z* direction.

Once the calibration was completed, the same equivalent shear modulus was used to compute the stiffness coefficients for the remaining degrees of freedom, and for the stiffness matrices of the foundations of the terminal viaducts, that have similar foundation soils.

Details of these calculations are reported in the Appendix B. Inspection of the calculation details for the Calabria Tower reveals that, once the shear modulus is chosen to match the vertical displacements, the agreement between the FEM and the elastic calculations for the horizontal displacement and for the rotation is not good: this is due to the sloping contact between the Pezzo Conglomerate and the upper deposits, that makes a model based on an elastic continuum hardly representative of the actual behaviour of the foundation.

For the anchor blocks, finite element analyses were carried out under plane strain conditions, using the same non-linear soil model used for the tower foundations. In this case, a preliminary equivalence was necessary to convert the displacements computed in plane strain to more realistic 3D displacements. This procedure is reported in A.7 and essentially consists in using elasticity theory to find the relationship between the horizontal displacement of an embedded strip and that of an equivalent embedded circular foundation.

Then, a procedure similar to that illustrated for the tower foundation was employed, in which the optimisation was carried out varying *G* and the coupling stiffness terms $k_{y\text{-}rx}$,

$k_{x\text{-}ry}$ to match the set of forces/displacements computed with FEM. For the anchor blocks, priority was given to matching forces/displacements in the *y* direction.

4 Stiffness And Compliance Matrices

Table 1 reports the stiffness matrices for the reference points provided by the structural analysts for the tower foundations, the anchor blocks, and the foundations of the terminal viaducts. Units are kN and m. Table 2 reports the corresponding compliance matrices, obtained by inversion of the stiffness matrices.

References

Gazetas G. (1991). Foundation vibrations. In: *Foundation Engineering Handbook, 2nd edition*, H.-Y. Fang, ed., Van Nostrand Reinhold, New York, 553-593.

Table 1. Stiffness matrices

Sicily Tower

	x	y	z	rx	ry	rz
x	3.2E+07	0	0	0	-3.5E+08	0
y	0	3.2E+07	0	-3.5E+08	0	0
z	0	0	2.9E+07	0	0	0
rx	0	-3.5E+08	0	2.0E+10	0	0
ry	-3.5E+08	0	0	0	2.0E+10	0
rz	0	0	0	0	0	3.1E+10

Units

x x	kN/m
y y	kN/m
z z	kN/m
rx rx	kN m
ry ry	kN m
rz rz	kN m
x ry	kN
y rx	kN

Calabria Tower

	x	y	z	rx	ry	rz
x	1.2E+08	0	0	0	-4.8E+08	0
y	0	1.2E+08	0	-4.8E+08	0	0
z	0	0	1.0E+08	0	0	0
rx	0	-4.8E+08	0	5.4E+10	0	0
ry	-4.8E+08	0	0	0	5.4E+10	0
rz	0	0	0	0	0	1.0E+11

Sicily Anchor

	x	y	z	rx	ry	rz
x	8.3E+07	0	0	0	1.8E+08	0
y	0	8.5E+07	0	1.8E+08	0	0
z	0	0	9.3E+07	0	0	0
rx	0	1.8E+08	0	1.6E+11	0	0
ry	1.8E+08	0	0	0	2.2E+11	0
rz	0	0	0	0	0	3.8E+11

Calabria Anchor

	x	y	z	rx	ry	rz
x	3.7E+08	0	0	0	-9.8E+08	0
y	0	3.8E+08	0	-9.9E+08	0	0
z	0	0	3.9E+08	0	0	0
rx	0	-9.9E+08	0	8.4E+11	0	0
ry	-9.8E+08	0	0	0	1.1E+12	0
rz	0	0	0	0	0	2.1E+12

Sicily terminal structure

	x	y	z	rx	ry	rz
x	2.1E+07	0	0	0	-1.5E+08	0
y	0	2.2E+07	0	-1.5E+08	0	0
z	0	0	1.9E+07	0	0	0
rx	0	-1.5E+08	0	9.8E+10	0	0
ry	-1.5E+08	0	0	0	1.3E+11	0
rz	0	0	0	0	0	1.7E+11

Calabria terminal structure

	x	y	z	rx	ry	rz
x	5.3E+07	0	0	0	1.6E+08	0
y	0	5.6E+07	0	1.7E+08	0	0
z	0	0	5.6E+07	0	0	0
rx	0	1.7E+08	0	3.0E+11	0	0
ry	1.6E+08	0	0	0	3.9E+11	0
rz	0	0	0	0	0	5.6E+11

Table 2. Compliance matrices

Sicily Tower

	x	y	z	rx	ry	rz
x	3.9E-08	0	0	0	6.7E-10	0
y	0	3.9E-08	0	6.7E-10	0	0
z	0	0	3.5E-08	0	0	0
rx	0	6.7E-10	0	6.1E-11	0	0
ry	6.7E-10	0	0	0	6.1E-11	0
rz	0	0	0	0	0	3.2E-11

Units

x x	m/KN
y y	m/KN
z z	m/KN
rx rx	1/(kN m)
ry ry	1/(kN m)
rz rz	1/(kN m)
x ry	1/kN
y rx	1/kN

Calabria Tower

	x	y	z	rx	ry	rz
x	8.4E-09	0	0	0	7.5E-11	0
y	0	8.4E-09	0	7.5E-11	0	0
z	0	0	9.7E-09	0	0	0
rx	0	7.5E-11	0	1.9E-11	0	0
ry	7.5E-11	0	0	0	1.9E-11	0
rz	0	0	0	0	0	9.6E-12

Sicily Anchor

	x	y	z	rx	ry	rz
x	1.2E-08	0	0	0	-9.7E-12	0
y	0	1.2E-08	0	-1.3E-11	0	0
z	0	0	1.1E-08	0	0	0
rx	0	-1.3E-11	0	6.1E-12	0	0
ry	-9.7E-12	0	0	0	4.6E-12	0
rz	0	0	0	0	0	2.6E-12

Calabria Anchor

	x	y	z	rx	ry	rz
x	2.7E-09	0	0	0	2.4E-12	0
y	0	2.7E-09	0	3.1E-12	0	0
z	0	0	2.6E-09	0	0	0
rx	0	3.1E-12	0	1.2E-12	0	0
ry	2.4E-12	0	0	0	9.1E-13	0
rz	0	0	0	0	0	4.8E-13

Sicily terminal structure

	x	y	z	rx	ry	rz
x	4.9E-08	0	0	0	5.7E-11	0
y	0	4.7E-08	0	7.3E-11	0	0
z	0	0	5.2E-08	0	0	0
rx	0	7.3E-11	0	1.0E-11	0	0
ry	5.7E-11	0	0	0	8.1E-12	0
rz	0	0	0	0	0	5.9E-12

Calabria terminal structure

	x	y	z	rx	ry	rz
x	1.9E-08	0	0	0	-7.7E-12	0
y	0	1.8E-08	0	-1.0E-11	0	0
z	0	0	1.8E-08	0	0	0
rx	0	-1.0E-11	0	3.3E-12	0	0
ry	-7.7E-12	0	0	0	2.6E-12	0
rz	0	0	0	0	0	1.8E-12

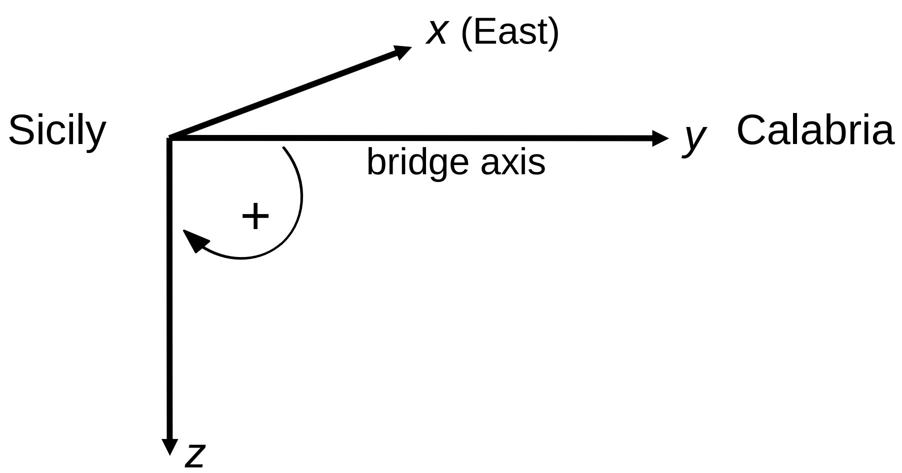

Figure 1. Frame of reference used in the calculations.

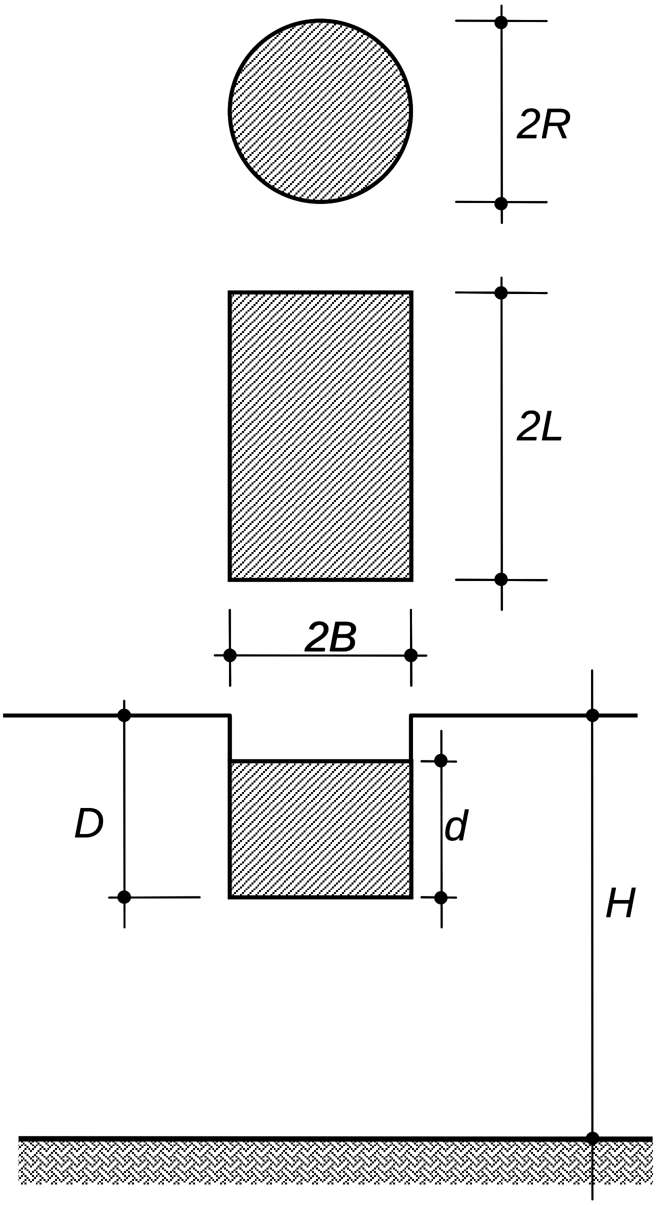

Figure 2. Schematic layout of an embedded rectangular foundation embedded in and elastic half-space.

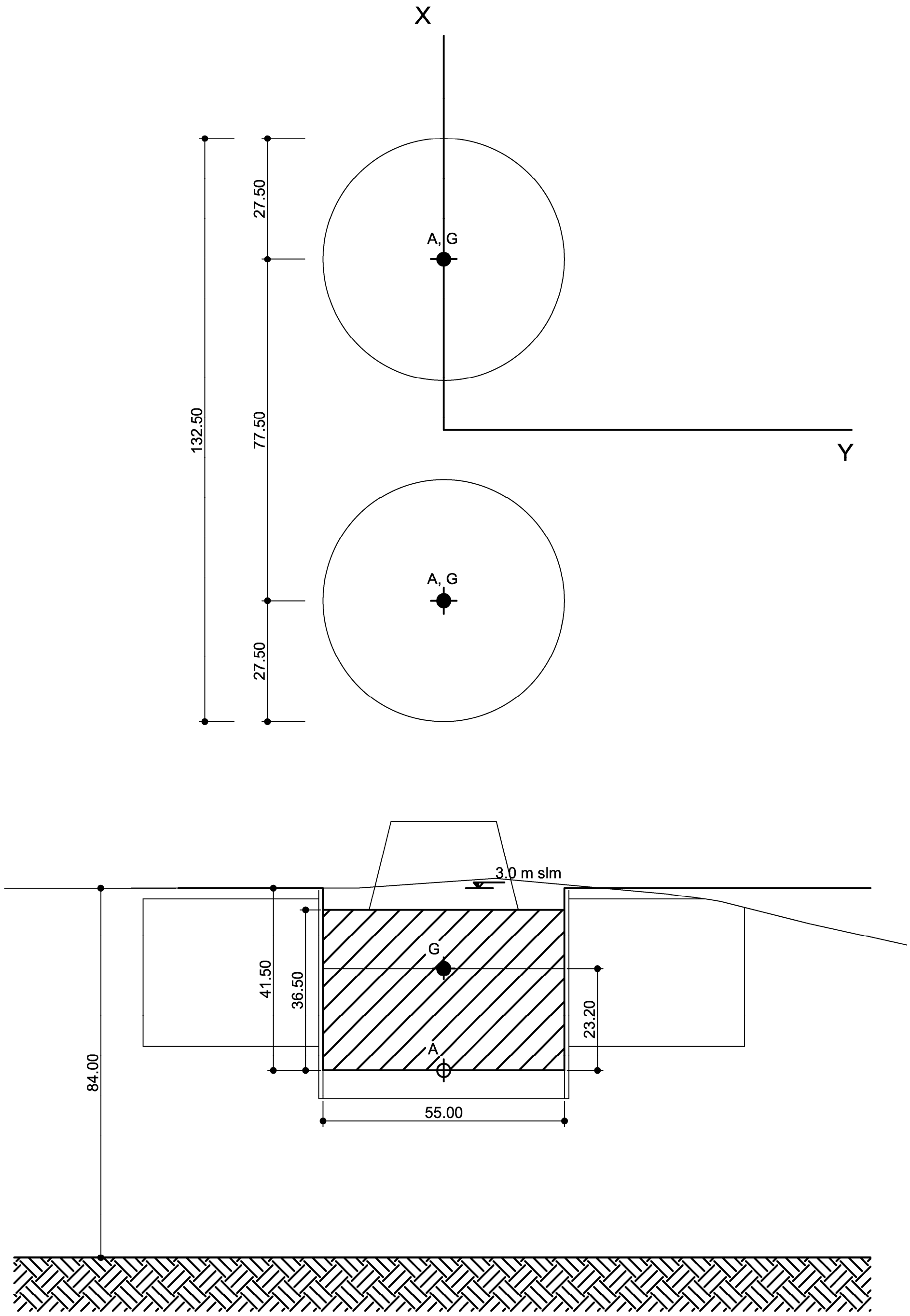

Figure 3. Foundation of the Sicily Tower.

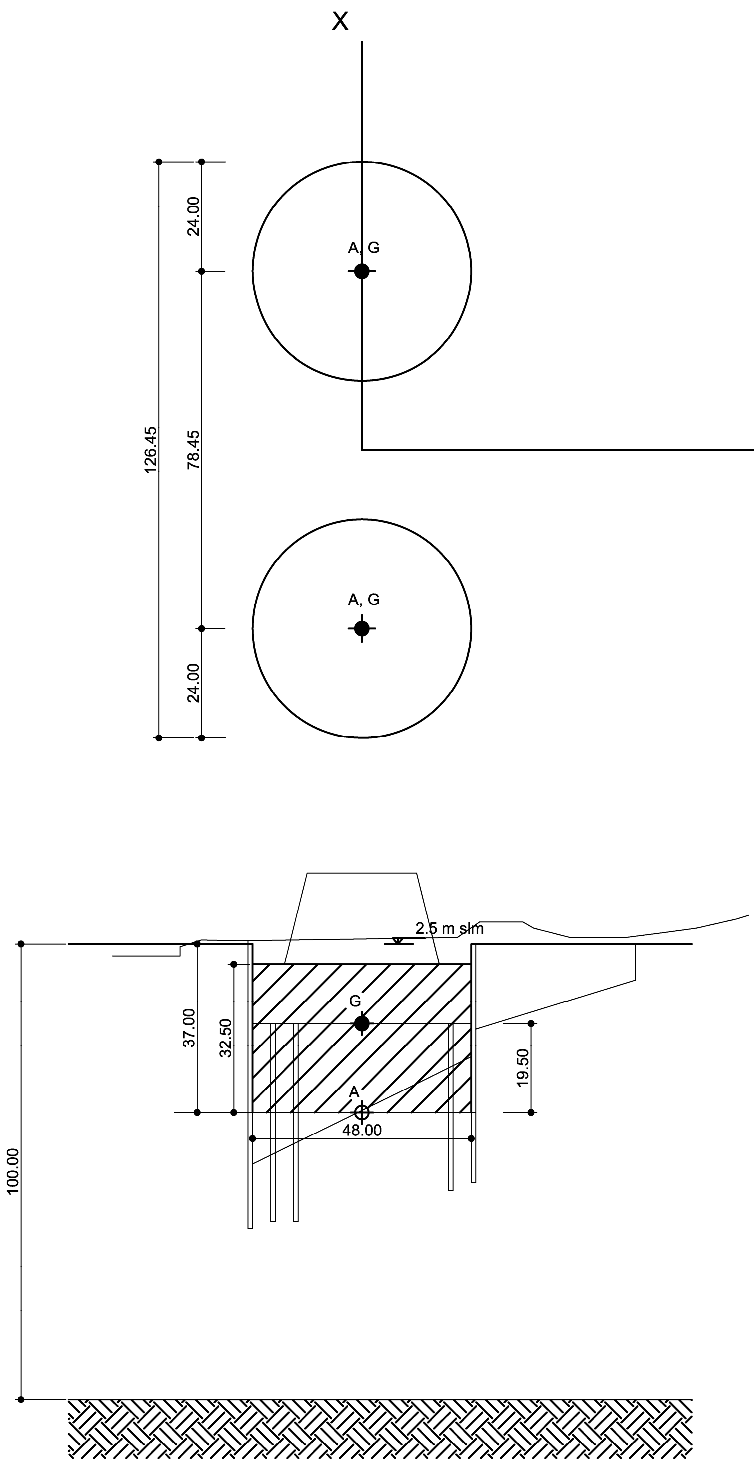

Figure 4. Foundation of the Calabria Tower.

Figure 5. Foundation of the Sicily anchor block.

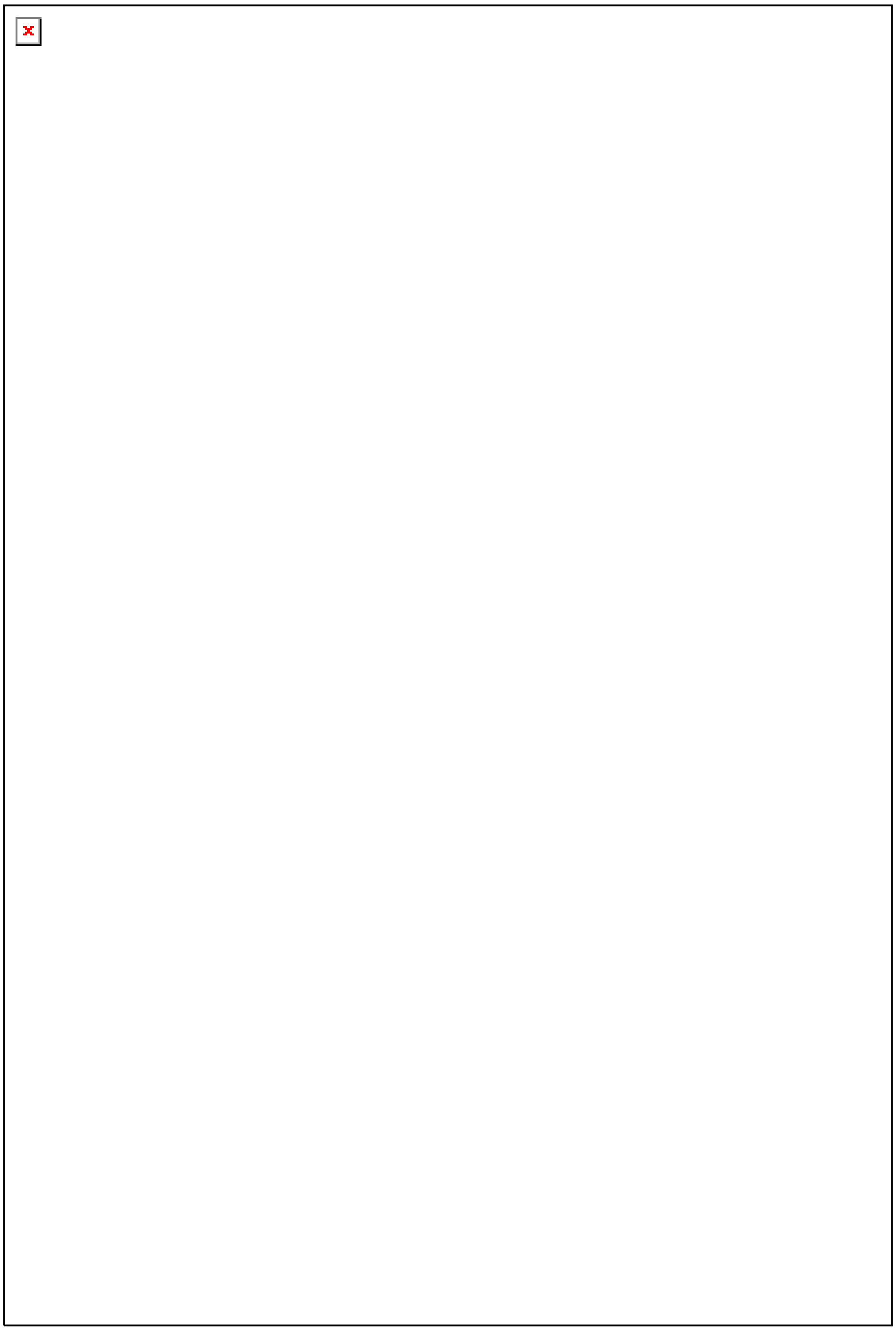

Figure 6. Foundation of the Calabria anchor block.

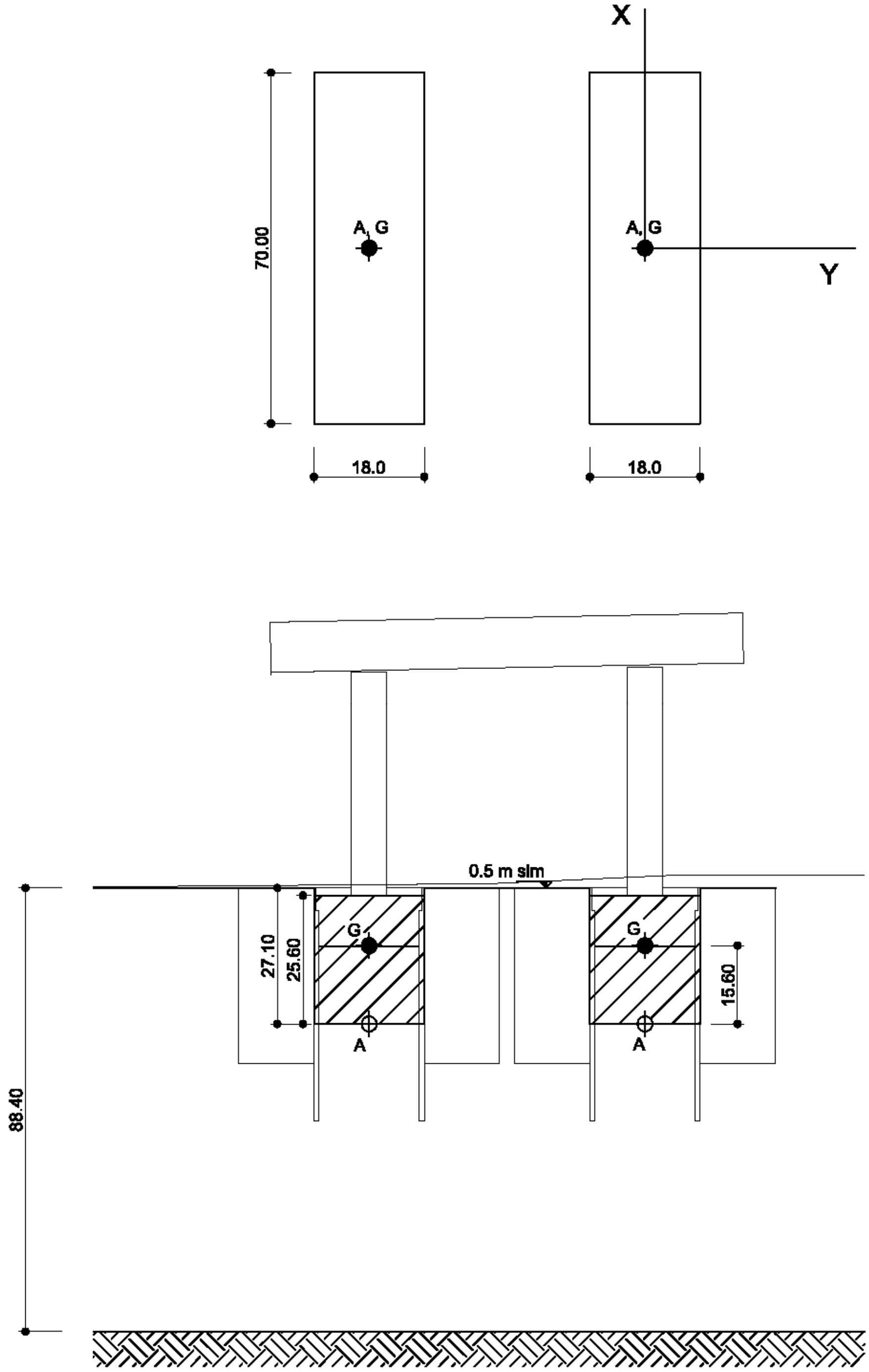

Figure 7. Foundations of the Sicilia terminal viaduct.

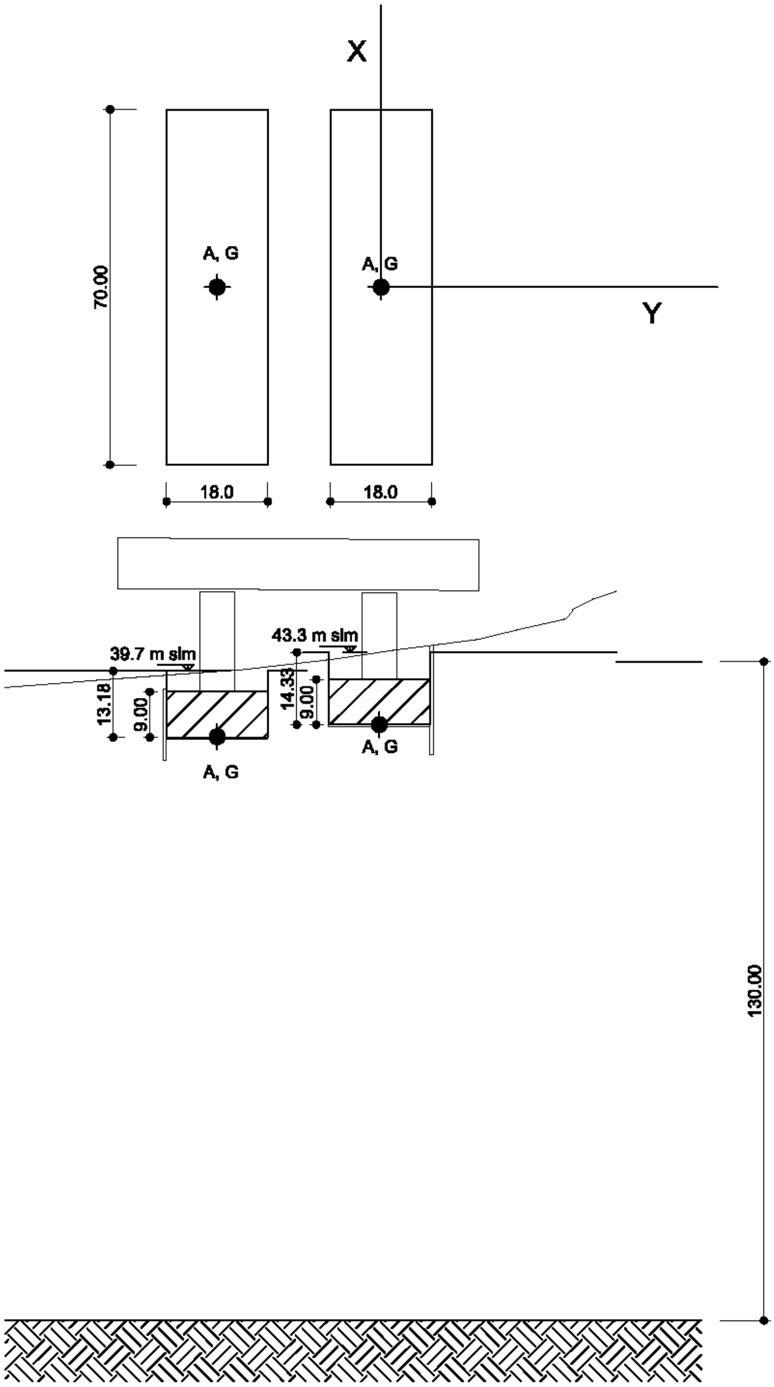

Figure 8. Foundations of the Calabria terminal viaduct.

APPENDIX A

Expressions for the elastic stiffness coefficients

1. Vertical translation: z direction

1. a Shallow rectangular foundation on elastic layer

$$k_{z(\text{shallow-r})} = \frac{2GL}{1-\nu}\left[0.73 + 1.54(\chi)^{3/4}\right]\cdot\left(1 + \frac{B/H}{0.5+\chi}\right)$$

$$\chi = \frac{A_b}{4L^2}$$

1.b Shallow circular foundation on elastic layer

$$k_{z(\text{shallow-c})} = \frac{4GR}{1-\nu}\left(1 + 1.3\frac{R}{H}\right)$$

1.c Equivalence for H $\rightarrow \infty$

$$R_{eq(z)} = \frac{L}{2}\left[0.73 + 1.54(\chi)^{3/4}\right]$$

1.d Embedded equivalent circular foundation on elastic layer

$$k_{z(\text{emb-c})} = k_{z(\text{shallow-c})}\left(1 + 0.55\frac{d}{R_{eq(z)}}\right)\cdot\left[1 + \left(0.85 - 0.28\frac{D}{R_{eq(z)}}\right)\cdot\frac{D}{H-D}\right]$$

1.e Embedded rectangular foundation on elastic half-space (for comparison)

$$k_{z(\text{emb-r})} = k_{z(\text{shallow-r})}\left[1 + \frac{1}{21}\cdot\frac{D}{B}(1+1.3\chi)\right]\cdot\left[1 + 0.2\left(\frac{A_w}{A_b}\right)^{2/3}\right]$$

$$A_w = perimeter \cdot d$$

2. Horizontal translation: y direction

2. a Shallow rectangular foundation on elastic half-space

$$k_{y(\text{shallow-r})} = \frac{2GL}{2-\nu}\left(2+2.5\chi^{0.85}\right)$$
$$\chi = \frac{A_b}{4L^2}$$

2. b Shallow circular foundation on elastic layer

$$k_{y(\text{shallow-c})} = \frac{8GR}{2-\nu}\left(1+0.5\frac{R}{H}\right)$$

2.c Equivalence for H $\rightarrow \infty$

$$R_{eq(y)} = \frac{L}{4}\left(2+2.5\chi^{0.85}\right)$$

2.d Embedded equivalent circular foundation on elastic layer

$$k_{y(\text{emb-c})} = k_{y(\text{shallow-c})}\left(1+\frac{d}{R_{eq(y)}}\right)\cdot\left(1+1.25\frac{D}{H}\right)$$

2.e Embedded rectangular foundation on elastic half-space (for comparison)

$$k_{y(\text{emb-r})} = k_{y(\text{shallow-r})}\left(1+0.15\sqrt{\frac{D}{B}}\right)\cdot\left[1+0.52\left(\frac{d}{B}\cdot\frac{A_w}{L^2}\right)^{0.4}\right]$$
$$A_w = perimeter \cdot d$$

3. Horizontal translation: x direction

3. a Shallow rectangular foundation on elastic half-space

$$k_{x(\text{shallow-r})} = k_{y(\text{shallow-r})} - \frac{0.2GL}{0.75-\nu}(1-\chi)$$

3. b Shallow circular foundation on elastic layer

$$k_{x(\text{shallow-c})} = k_{y(\text{shallow-c})} = \frac{8GR}{2-\nu}\left(1+0.5\frac{R}{H}\right)$$

3.c Equivalence for H $\rightarrow \infty$

$$R_{eq(x)} = \frac{L}{8}\left[2\cdot\left(2+2.5\chi^{0.85}\right) - \frac{0.2\cdot(2-\nu)}{0.75-\nu}(1-\chi)\right]$$

3.d Embedded equivalent circular foundation on elastic layer

$$k_{x(\text{emb-c})} = k_{y(\text{emb-c})} = k_{y(\text{shallow-c})}\left(1+\frac{d}{R_{eq(x)}}\right)\cdot\left(1+1.25\frac{D}{H}\right)$$

3.e Embedded rectangular foundation on elastic half-space (for comparison)

$$k_{x(\text{emb-r})} = k_{x(\text{shallow-r})}\left(1+0.15\sqrt{\frac{D}{B}}\right)\cdot\left[1+0.52\left(\frac{d}{B}\cdot\frac{A_w}{L^2}\right)^{0.4}\right]$$

$$A_w = perimeter \cdot d$$

4. Rotation around x axis

4.a Shallow rectangular foundation on elastic half-space

$$k_{rx(\text{shallow-r})} = \frac{G}{1-\nu} \cdot I_{bx}^{0.75} \left(\frac{L}{B}\right)^{0.25} \left(2.4 + 0.5\frac{B}{L}\right)$$

I_{bx} = *area moment of inertia of the foundation – soil contact surface around the x axis*

$I_{bx} = 2\frac{\pi R^4}{4}$ *for Tower foundations*

$I_{bx} = 2L\frac{(2B)^3}{12}$ *for Anchor Blocks*

4.b Shallow circular foundation on elastic layer

$$k_{rx(\text{shallow-c})} = \frac{8GR^3}{3(1-\nu)} \cdot \left(1 + 0.17\frac{R}{H}\right)$$

4.c Equivalence for H $\rightarrow \infty$

$$R_{eq(rx)} = \left[\frac{3}{8} I_{bx}^{0.75} \left(\frac{L}{B}\right)^{0.25} \left(2.4 + 0.5\frac{B}{L}\right)\right]^{1/3}$$

4.d Embedded equivalent circular foundation on elastic layer

$$k_{rx(\text{emb-c})} = k_{rx(\text{shallow-c})} \cdot \left(1 + 2\frac{d}{R_{eq(rx)}}\right) \cdot \left(1 + 0.65\frac{D}{H}\right)$$

4.e Embedded rectangular foundation on elastic half-space (for comparison)

$$k_{rx(\text{emb-r})} = k_{rx(\text{shallow-r})} \cdot \left\{1 + 1.26\frac{d}{B}\left[1 + \frac{d}{B}\left(\frac{d}{B}\right)^{-0.2}\sqrt{\frac{B}{L}}\right]\right\}$$

5. Rotation around axis y

5.a Shallow rectangular foundation on elastic half-space

$$k_{ry(\text{shallow-r})} = \frac{G}{1-\nu} \cdot I_{by}{}^{0.75} 3\left(\frac{L}{B}\right)^{0.15}$$

I_{by} = *area moment of inertia of the foundation – soil contact surface around the y axis*

$$I_{by} = 2\left[\frac{\pi R^4}{4} + \pi R^2 \left(\frac{s}{2}\right)^2\right]$$ *for Tower foundations with s= spacing between footings*

$$I_{bx} = 2B\frac{(2L)^3}{12}$$ *for Anchor Blocks*

5.b Shallow circular foundation on elastic layer

$$k_{ry(\text{shallow-c})} = k_{rx(\text{shallow-c})} = \frac{8GR^3}{3(1-\nu)} \cdot \left(1 + 0.17\frac{R}{H}\right)$$

5.c Equivalence for H → ∞

$$R_{eq(ry)} = \left[\frac{3}{8} I_{by}{}^{0.75} 3\left(\frac{L}{B}\right)^{0.15}\right]^{1/3}$$

5.d Embedded equivalent circular foundation on elastic layer

$$k_{ry(\text{emb-c})} = k_{ry(\text{shallow-c})} \cdot \left(1 + 2\frac{d}{R_{eq(ry)}}\right) \cdot \left(1 + 0.65\frac{D}{H}\right)$$

5.e Embedded rectangular foundation on elastic half-space (for comparison)

$$k_{ry(\text{emb-r})} = k_{ry(\text{shallow-r})} \cdot \left\{1 + 0.92\left(\frac{d}{L}\right)^{0.6}\left[1.5 + \left(\frac{d}{L}\right)^{1.9}\left(\frac{d}{D}\right)^{-0.6}\right]\right\}$$

6. Rotation around vertical z axis

6.a Shallow rectangular foundation on elastic half-space

$$k_{rz(\text{shallow-r})} = G \cdot I_b^{0.75}\left[4 + 11\left(1 - \frac{B}{L}\right)^{10}\right]$$

$I_b = I_{bx} + I_{by}$ = *polar moment of inertia of the soil – foundation contact surface*

6.b Shallow circular foundation on elastic layer

$$k_{rz(\text{shallow-c})} = \frac{16GR^3}{3} \cdot \left(1 + 0.10\frac{R}{H}\right)$$

6.c Equivalence for H $\rightarrow \infty$

$$R_{eq(rz)} = \left\{\frac{3}{16} I_b^{0.75}\left[4 + 11 \cdot \left(1 - \frac{B}{L}\right)^{10}\right]\right\}^{1/3}$$

6.d Embedded equivalent circular foundation on elastic layer

$$k_{rz(\text{emb-c})} = k_{rz(\text{shallow-c})} \cdot \left(1 + 2.67\frac{d}{R_{eq(rz)}}\right)$$

6.e Embedded rectangular foundation on elastic half-space (for comparison)

$$k_{rz(\text{emb-r})} = k_{rz(\text{shallow-r})}\left[1 + 1.4 \cdot \left(1 + \frac{B}{L}\right) \cdot \left(\frac{d}{B}\right)^{0.9}\right]$$

7. 2D versus 3D equivalence (used for Anchor Blocks)

7.a Embedded strip foundation on elastic layer

$$k_{y(\text{emb-strip})}{}^{2D} = \frac{0.73G}{1-\nu}\left(1+3.5\frac{B}{H}\right)\cdot\left(1+0.5\frac{d}{B}\right)\cdot\left(1+1.5\frac{D}{H}\right)$$

displacement: $u^{2D} = \dfrac{T^{2D}}{k_{y(\text{emb-strip})}{}^{2D}}$

with $T^{2D} = T^{3D}/L$ and L = 100 m

7.b Embedded equivalent circular foundation on elastic layer

$$k_{y(\text{emb-c})}{}^{3D} = k_{y(\text{shallow-c})}\left(1+\frac{d}{R_{eq}}\right)\cdot\left(1+1.25\frac{D}{H}\right)$$

displacement: $u^{3D} = \dfrac{T^{3D}}{k_{y(\text{emb-c})}{}^{3D}}$

7.c Displacement correction factor

$$\frac{u^{3D}}{u^{2D}} = \frac{T^{3D}}{T^{2D}}\cdot\frac{k_{y(\text{emb-strip})}{}^{2D}}{k_{y(\text{emb-c})}{}^{3D}} = L\cdot\frac{k_y{}^{2D}}{k_y{}^{3D}} = \mu$$

$$u^{3D} = \mu\cdot u^{2D}$$

APPENDIX B

Tables for the evaluation of the stiffness coefficients

SICILY TOWER single footing

ELASTIC SOLUTIONS

single footing					
radius	R	27.5 m			
total area	Ab	m2			
distance	i	77.5 m	γ foundation	18.4	kN/m3
equivalent rectangle			Δh foundation	2.5	m
			ΔN @ 2.5 m o.d.	218.6	MN
			ΔN @ -15.5 m o.d.	2318.6	MN
embedment	D	41.5 m	level, l	5	m o.d.
	d	36.5 m	ground level	2.5	m o.d.
			base level	-38.5	m o.d.
			difference	43.5	m
thickness of layer	H	84 m			
shear modulus	G	**60** MPa			
Poisson's ratio	ν	**0.2**			
Young's modulus	E	144.0 MPa			

FEM 3D (to calibrate elastic stiffness G)

	N	H	M
	z	y	rx
	MN	MN	MN m
loads @ level l	1288.5	185	10973
loads @ -15.5 m o.d.	1507	185	19020.5
	m	m	rad
FEM displacements	0.056	-0.004	0.0008
back-computed forces	1615	184	15396
ratios (computed/FEM)	**1.07**	**1.00**	**0.81**
back-computed displ.	0.05	-0.01	0.0010
ratios (computed/FEM)	**0.93**	**1.66**	**1.27**

vertical displacement (z)

shallow circular found. on elastic layer	Kz(shallow-c)	11761.2 MN m	
embedded foundation	Kz (emb-c)	**2.88E+04 MN/m**	**2.88E+07 kN/m**

torsion (rz)

shallow circular found. on elastic layer	Krz(shallow-c)	6872872.02 MN m	
embedded circular found. on elastic layer	Krz(emb-c)	**3.12E+07 MN m**	**3.12E+10 kN m**

horizontal displacement (y)
(along the bridge)

shallow circular found. on elastic layer	Ky(shallow-c)	8533.73016 MN/m	
embedded circular found. on elastic layer	Ky(emb-c)	**32125.2457 MN/m**	**3.21E+07 kN/m**

horizontal displacement (x)
(orthogonal to the bridge)

shallow circular found. on elastic layer	Kx(shallow-c)	8533.73016	
embedded circular found. on elastic layer	Kx(emb-c)	**32125.2457 MN/m**	**3.21E+07 kN/m**

rotation around x (rx)

shallow circular found. on elastic layer	Krx(shallow-c)	4390864.03 MN m	
embedded circular found. on elastic layer	Krx(emb-c)	**21199676 MN m**	**2.12E+10 kN m**

rotation around y (ry)

shallow circular found. on elastic layer	Kry(shallow-c)	4390864.03	
embedded circular found. on elastic layer	Kry(emb-c)	**21199676 MN m**	**2.12E+10 kN m**

mixed mode

circular y-rx	Ky-rx	**390857.155 MN**	**3.91E+08 kN**

mixed mode

circular x-ry	Kx-ry	**390857.155 MN**	**3.91E+08 kN**

CALABRIA TOWER single footing

ELASTIC SOLUTIONS

single footing							
radius	R	24	m				
total area	Ab	3619.1	m2				
distance	i	78.45	m	γ foundation	18.7	kN/m3	
equivalent rectangle				Δh foundation	15.5	m	
				ΔN @ 5 m o.d.	1049	MN	
				ΔN @ -15.5 m o.d.	2319	MN	
embedment	D	37	m	level, l	7	m o.d.	
	d	32.5	m	ground level	2.5	m o.d.	
				base level	-29.7	m o.d.	
				difference	36.7	m	
thickness of layer	H	100	m				
shear modulus	G	**300**	MPa	350			
Poisson's ratio	ν	**0.2**					
Young's modulus	E	720.0	MPa				

FEM 3D (to calibrate elastic stiffness G)

	N	H	M
	z	y	rx
	MN	MN	MN m
loads @ level l	1296.5	-178	-12761
loads @ -15.5 m o.d.	2346	-178	-6228.4
	m	m	rad
FEM displacements	0.0194	-0.026	-0.0006
back-computed forces	1992	-4064	-77117
ratios (computed/FEM)	**0.85**	**22.83**	**12.38**
back-computed displ.	0.02	0.00	-0.0001
ratios (computed/FEM)	**1.18**	**0.02**	**0.13**

vertical displacement (z)

shallow circular found. on elastic layer	Kz(shallow-c)	47232.0	MN m	
embedded foundation	Kz (emb-c)	**1.03E+05**	**MN/m**	**1.03E+08 kN/m**

torsion (rz)

shallow circular found. on elastic layer	Krz(shallow-c)	22649241.6	MN m	
embedded circular found. on elastic layer	Krz(emb-c)	**1.05E+08**	**MN m**	**1.05E+11 kN m**

horizontal displacement (y)
(along the bridge)

shallow circular found. on elastic layer	Ky(shallow-c)	35840	MN/m	
embedded circular found. on elastic layer	Ky(emb-c)	**123396**	**MN/m**	**1.23E+08 kN/m**

horizontal displacement (x)
(orthogonal to the bridge)

shallow circular found. on elastic layer	Kx(shallow-c)	35840		
embedded circular found. on elastic layer	Kx(emb-c)	**123396**	**MN/m**	**1.23E+08 kN/m**

rotation around x (rx)

shallow circular found. on elastic layer	Krx(shallow-c)	14388019.2	MN m	
embedded circular found. on elastic layer	Krx(emb-c)	**66187586.1**	**MN m**	**6.62E+10 kN m**

rotation around y (ry)

shallow circular found. on elastic layer	Kry(shallow-c)	14388019.2		
embedded circular found. on elastic layer	Kry(emb-c)	**66187586.1**	**MN m**	**6.62E+10 kN m**

mixed mode

y-rx	Ky-rx	**1336790**	**MN**	**1.34E+09 kN**

mixed mode

x-ry	Kx-ry	**1336790**	**MN**	**1.34E+09 kN**

SICILY ANCHOR BLOCK

ELASTIC SOLUTIONS

total area	Ab	10685	m2
equivalent rectangle			
half-width	B	44	m
half-length	L	64.5	m
	χ	0.642	
		1.46591	
embedment	D	43.65	m
	d	36	m
	Aw	14940	m2
thickness of layer	H	145	m
	B/H	0.303	
shear modulus	G	**60**	MPa
Poisson's ratio	ν	**0.2**	
Young's modulus	E	144.0	MPa

2D/3D equivalence
L (m)
100
factor (see 7.a)
0.54

FEM 2D (to calibrate elastic stiffness G)

	N	**H**	**M**
	z	y	rx
	MN	MN	MN m
Loads @ reference point	-1026	3829	0
Loads @ found. centre/bott	-1026	3829	119909
	m	m	rad
2D displacements	-0.042	0.167	0.0008
equivalent 3D displ.s	**-0.023**	**0.091**	**0.0004**
ratios (computed/FEM)	**1.03**	**1.11**	**1.05**
comp. forces @ centre/bott.	-1057	4248	125624
computed displacements	-0.02	0.08	0.0005
ratio (computed/FEM)	**0.97**	**0.88**	**1.09**

vertical displacement (z)

shallow rectangular found. on elastic half-space	Kz (shallow-r)	22466.2	MN/m	
equivalent radius	Req(z)	59.2	m	
shallow circular found. on elastic layer	Kz(shallow-c)	27165.8	MN m	
embedded foundation	Kz (emb-c)	**4.63E+04**	**MN/m**	**4.63E+07 kN/m**
embedded rectang. found. on elastic half-space	Kz (emb-r)	**3.05E+04**	**MN/m**	**3.05E+07 kN/m**

torsion (rz)

polar moment of inertia	Ib=Ibx+Iby	19356817	m4	
shallow foundation on elastic half-space	Krz(shallow-r)	70040503.9	MN m	
equivalent radius	Req(rz)	60.3	m	
shallow circular found. on elastic layer	Krz(shallow-c)	72951540.6	MN m	
embedded circular found. on elastic layer	Krz(emb-c)	**1.89E+08**	**MN m**	**1.89E+11 kN m**
embedded rectang. found. on elastic half-space	Krz(emb-r)	**2.08E+08**	**MN m**	**2.08E+11 kN m**

horizontal displacement (y)
(along the bridge)

shallow rectangular found. on elastic half-space	Ky(shallow-r)	15976.7371	MN/m	
equivalent radius	Req(y)	59.9	m	
shallow circular found. on elastic layer	Ky(shallow-c)	19277.4629	MN/m	
embedded circular found. on elastic layer	Ky(emb-c)	**4.25E+04**	**MN/m**	**4.25E+07 kN/m**
embedded rectang. found. on elastic half-space	Ky(emb-r)	**3.31E+04**	**MN/m**	**3.31E+07 kN/m**
embeddede strip foundation on elastic layer	Ky(emb-strip)	**230.918559**	**MN/m2**	

horizontal displacement (x)
(trasversale ponte)

shallow rectangular found. on elastic half-space	Kx(shallow-r)	15473.0584		
equivalent radius	Req(x)	58.0	m	
shallow circular found. on elastic layer	Kx(shallow-c)	18568.949		
embedded circular found. on elastic layer	Kx(emb-c)	**4.14E+04**	**MN/m**	**4.14E+07 kN/m**
embedded rectang. found. on elastic half-space	Kx(emb-r)	**3.20E+04**	**MN/m**	**3.20E+07 kN/m**

rotation around x (rx)

moment of inertia	Ibx	8186419	m4	
shallow foundation on elastic half-space	Krx(shallow-r)	34620317.4	MN m	
equivalent radius	Req(rx)	55.7	m	
shallow circular found. on elastic layer	Krx(shallow-c)	36882420.1	MN m	
embedded circular found. on elastic layer	Krx(emb-c)	**1.01E+08**	**MN m**	**1.01E+11 kN m**
embedded rectang. found. on elastic half-space	Krx(emb-r)	**1.06E+08**	**MN m**	**1.06E+11 kN m**

rotation around y (ry)

moment of inertia	Iby	11170398	m4	
shallow foundation on elastic half-space	Kry(shallow-r)	46041563.1	MN m	
equivalent radius	Req(ry)	61.3	m	
shallow circular found. on elastic layer	Kry(shallow-c)	49349858.9		
embedded circular found. on elastic layer	Kry(emb-c)	**1.28E+08**	**MN m**	**1.28E+11 kN m**
embedded rectang. found. on elastic half-space	Kry(emb-r)	**1.02E+08**	**MN m**	**1.02E+11 kN m**

mixed mode

circular y-rx	Ky-rx	**8.99E+05**	**MN**	**8.99E+08 kN**

mixed mode

circular x-ry	Kx-ry	**8.77E+05**	**MN**	**8.77E+08 kN**

CALABRIA ANCHOR BLOCK

ELASTIC SOLUTIONS

total area	Ab	8950	m2
equivalent rectangle			
half-width	B	44.75	m
half-length	L	50	m
	χ	0.895	
embedment	D	37	m
	d	37	m
	Aw	15355	m2
thickness of layer	H	180	m
	B/H	0.249	
shear modulus	G	**400**	MPa
Poisson's ratio	ν	**0.2**	
Young's modulus	E	960.0	MPa

2D/3D equivalence
L (m)
100
factor (see 7.a)
0.54

FEM 2D (to calibrate elastic stiffness G)

	N	**H**	**M**
	z	y	rx
	MN	MN	MN m
Loads @ reference point	-1018	-3800	0
Loads @ found. centre/bott.	-1018	-3800	-120425
	m	m	rad
2D displacements	-0.004	-0.030	-0.0001
equivalent 3D displ.s	**-0.002**	**-0.016**	**-2.72E-05**
ratios (computed/FEM)	**0.57**	**1.03**	**0.83**
comp. forces @ centre/bott.	-575	-3924	-99429
computed displacements	0.00	-0.01	-0.0001
ratio (computed/FEM)	**1.77**	**0.91**	**2.69**

vertical displacement (z)

shallow rectangular found. on elastic half-space	Kz (shallow-r)	126485.0 MN/m	
equivalent radius	Req(z)	53.7 m	
shallow circular found. on elastic layer	Kz(shallow-c)	148969.8 MN m	
embedded foundation	Kz (emb-c)	**2.40E+05 MN/m**	**2.40E+08 kN/m**
embedded rectang. found. on elastic half-space	Kz (emb-r)	**1.77E+05 MN/m**	**1.77E+08 kN/m**

torsion (rz)

polar moment of inertia	Ib=Ibx+Iby	19356817 m4	
shallow foundation on elastic half-space	Krz(shallow-r)	466923187 MN m	
equivalent radius	Req(rz)	60.3 m	
shallow circular found. on elastic layer	Krz(shallow-c)	482555929 MN m	
embedded circular found. on elastic layer	Krz(emb-c)	**1.27E+09 MN m**	**1.27E+12 kN m**
embedded rectang. found. on elastic half-space	Krz(emb-r)	**1.51E+09 MN m**	**1.51E+12 kN m**

horizontal displacement (y)
(along the bridge)

shallow rectangular found. on elastic half-space	Ky(shallow-r)	95000.9531 MN/m	
equivalent radius	Req(y)	53.4 m	
shallow circular found. on elastic layer	Ky(shallow-c)	109102.799 MN/m	
embedded circular found. on elastic layer	Ky(emb-c)	**2.32E+05 MN/m**	**2.32E+08 kN/m**
embedded rectang. found. on elastic half-space	Ky(emb-r)	**2.15E+05 MN/m**	**2.15E+08 kN/m**
embeddede strip foundatio on elastic layer	Ky(emb-strip)	**1262.27105 MN/m2**	

horizontal displacement (x)
(trasversale ponte)

shallow rectangular found. on elastic half-space	Kx(shallow-r)	94237.3168	
equivalent radius	Req(x)	53.0 m	
shallow circular found. on elastic layer	Kx(shallow-c)	108113.367	
embedded circular found. on elastic layer	Kx(emb-c)	**2.31E+05 MN/m**	**2.31E+08 kN/m**
embedded rectang. found. on elastic half-space	Kx(emb-r)	**2.14E+05 MN/m**	**2.14E+08 kN/m**

rotation around x (rx)

moment of inertia	Ibx	8186419 m4	
shallow foundation on elastic half-space	Krx(shallow-r)	224026016 MN m	
equivalent radius	Req(rx)	55.2 m	
shallow circular found. on elastic layer	Krx(shallow-c)	235701138 MN m	
embedded circular found. on elastic layer	Krx(emb-c)	**6.26E+08 MN m**	**6.26E+11 kN m**
embedded rectang. found. on elastic half-space	Krx(emb-r)	**6.40E+08 MN m**	**6.40E+11 kN m**

rotation around y (ry)

moment of inertia	Iby	11170398 m4	
shallow foundation on elastic half-space	Kry(shallow-r)	294692640 MN m	
equivalent radius	Req(ry)	60.5 m	
shallow circular found. on elastic layer	Kry(shallow-c)	311520254	
embedded circular found. on elastic layer	Kry(emb-c)	**7.85E+08 MN m**	**7.85E+11 kN m**
embedded rectang. found. on elastic half-space	Kry(emb-r)	**7.62E+08 MN m**	**7.62E+11 kN m**

mixed mode

circular y-rx	Ky-rx	**5.05E+06 MN**	**5.05E+09 kN**

mixed mode

circular x-ry	Kx-ry	**5.02E+06 MN**	**5.02E+09 kN**

SICILY TERMINAL STRUCTURE

ELASTIC SOLUTIONS

total area	Ab	1540	m2
equivalent rectangle			
half-width	B	11	m
half-length	L	35	m
	χ	0.314	
embedment	D	27.1	m
	d	25.6	m
	Aw	4710.4	m2
thickness of layer	H	88.4	m
	B/H	0.124	
shear modulus	G	**60**	MPa
Poisson's ratio	ν	**0.2**	
Young's modulus	E	144.0	MPa

vertical displacement (z)

shallow rectangular found. on elastic half-space	Kz (shallow-r)	8330.5 MN/m	
equivalent radius	Req(z)	24.1 m	
shallow circular found. on elastic layer	Kz(shallow-c)	9785.9 MN m	
embedded foundation	Kz (emb-c)	**1.92E+04 MN/m**	**1.92E+07 kN/m**
embedded rectang. found. on elastic half-space	Kz (emb-r)	**1.38E+04 MN/m**	**1.38E+07 kN/m**

torsion (rz)

polar moment of inertia	Ib=Ibx+Iby	19356817 m4	
shallow foundation on elastic half-space	Krz(shallow-r)	74465394.9 MN m	
equivalent radius	Req(rz)	61.5 m	
shallow circular found. on elastic layer	Krz(shallow-c)	79646675 MN m	
embedded circular found. on elastic layer	Krz(emb-c)	**1.68E+08 MN m**	**1.68E+11 kN m**
embedded rectang. found. on elastic half-space	Krz(emb-r)	**3.68E+08 MN m**	**3.68E+11 kN m**

horizontal displacement (y)
(along the bridge)

shallow rectangular found. on elastic half-space	Ky(shallow-r)	6847.60175 MN/m	
equivalent radius	Req(y)	25.7 m	
shallow circular found. on elastic layer	Ky(shallow-c)	7842.15032 MN/m	
embedded circular found. on elastic layer	Ky(emb-c)	**2.17E+04 MN/m**	**2.17E+07 kN/m**
embedded rectang. found. on elastic half-space	Ky(emb-r)	**1.90E+04 MN/m**	**1.90E+07 kN/m**
embeddede strip foundatioı on elastic layer	Ky(emb-strip)2D	**248.246707 MN/m2**	

horizontal displacement (x)
(trasversale ponte)

shallow rectangular found. on elastic half-space	Kx(shallow-r)	6323.96539	
equivalent radius	Req(x)	23.7 m	
shallow circular found. on elastic layer	Kx(shallow-c)	7172.22341	
embedded circular found. on elastic layer	Kx(emb-c)	**2.06E+04 MN/m**	**2.06E+07 kN/m**
embedded rectang. found. on elastic half-space	Kx(emb-r)	**1.76E+04 MN/m**	**1.76E+07 kN/m**

rotation around x (rx)

moment of inertia	Ibx	8186419 m4	
shallow foundation on elastic half-space	Krx(shallow-r)	39201760.5 MN m	
equivalent radius	Req(rx)	58.1 m	
shallow circular found. on elastic layer	Krx(shallow-c)	43580953.5 MN m	
embedded circular found. on elastic layer	Krx(emb-c)	**9.83E+07 MN m**	**9.83E+10 kN m**
embedded rectang. found. on elastic half-space	Krx(emb-r)	**3.22E+08 MN m**	**3.22E+11 kN m**

rotation around y (ry)

moment of inertia	Iby	11170398 m4	
shallow foundation on elastic half-space	Kry(shallow-r)	51717236.1 MN m	
equivalent radius	Req(ry)	63.7 m	
shallow circular found. on elastic layer	Kry(shallow-c)	58053507	
embedded circular found. on elastic layer	Kry(emb-c)	**1.26E+08 MN m**	**1.26E+11 kN m**
embedded rectang. found. on elastic half-space	Kry(emb-r)	**1.33E+08 MN m**	**1.33E+11 kN m**

mixed mode

circular y-rx	Ky-rx	**1.85E+05 MN**	**1.85E+08 kN**

mixed mode

circular x-ry	Kx-ry	**1.76E+05 MN**	**1.76E+08 kN**

CALABRIA TERMINAL STRUCTURE

ELASTIC SOLUTIONS

total area	Ab	1400	
equivalent rectangle			
half-width	B	10	
half-length	L	35	
	χ	0.286	
embedment	D	13.75	
	d	9	
	Aw	1620	
thickness of layer	H	130	
	B/H	0.077	
shear modulus	G	**300**	MPa
Poisson's ratio	ν	**0.2**	
Young's modulus	E	720.0	MPa

vertical displacement (z)

shallow rectangular found. on elastic half-space	Kz (shallow-r)	38383.1 MN/m	
equivalent radius	Req(z)	23.3 m	
shallow circular found. on elastic layer	Kz(shallow-c)	43108.6 MN m	
embedded foundation	Kz (emb-c)	**5.65E+04 MN/m**	**5.65E+07 kN/m**
embedded rectang. found. on elastic half-space	Kz (emb-r)	**5.11E+04 MN/m**	**5.11E+07 kN/m**

torsion (rz)

polar moment of inertia	Ib=Ibx+Iby	19356817 m4	
shallow foundation on elastic half-space	Krz(shallow-r)	383485858 MN m	
equivalent radius	Req(rz)	62.1 m	
shallow circular found. on elastic layer	Krz(shallow-c)	401809669 MN m	
embedded circular found. on elastic layer	Krz(emb-c)	**5.57E+08 MN m**	**5.57E+11 kN m**
embedded rectang. found. on elastic half-space	Krz(emb-r)	**1.01E+09 MN m**	**1.01E+12 kN m**

horizontal displacement (y)
(along the bridge)

shallow rectangular found. on elastic half-space	Ky(shallow-r)	33389.4189 MN/m	
equivalent radius	Req(y)	25.0 m	
shallow circular found. on elastic layer	Ky(shallow-c)	36605.3419 MN/m	
embedded circular found. on elastic layer	Ky(emb-c)	**5.63E+04 MN/m**	**5.63E+07 kN/m**
embedded rectang. found. on elastic half-space	Ky(emb-r)	**6.12E+04 MN/m**	**6.12E+07 kN/m**
embeddede strip foundatio on elastic layer	Ky(emb-strip)	**583.735935 MN/m2**	

horizontal displacement (x)
(trasversale ponte)

shallow rectangular found. on elastic half-space	Kx(shallow-r)	30662.1462	
equivalent radius	Req(x)	23.0 m	
shallow circular found. on elastic layer	Kx(shallow-c)	33374.167	
embedded circular found. on elastic layer	Kx(emb-c)	**5.26E+04 MN/m**	**5.26E+07 kN/m**
embedded rectang. found. on elastic half-space	Kx(emb-r)	**5.62E+04 MN/m**	**5.62E+07 kN/m**

rotation around x (rx)

moment of inertia	Ibx	8186419 m4	
shallow foundation on elastic half-space	Krx(shallow-r)	199613872 MN m	
equivalent radius	Req(rx)	58.4 m	
shallow circular found. on elastic layer	Krx(shallow-c)	214869374 MN m	
embedded circular found. on elastic layer	Krx(emb-c)	**3.00E+08 MN m**	**3.00E+11 kN m**
embedded rectang. found. on elastic half-space	Krx(emb-r)	**6.80E+08 MN m**	**6.80E+11 kN m**

rotation around y (ry)

moment of inertia	Iby	11170398 m4	
shallow foundation on elastic half-space	Kry(shallow-r)	262309618 MN m	
equivalent radius	Req(ry)	64.0 m	
shallow circular found. on elastic layer	Kry(shallow-c)	284267537	
embedded circular found. on elastic layer	Kry(emb-c)	**3.89E+08 MN m**	**3.89E+11 kN m**
embedded rectang. found. on elastic half-space	Kry(emb-r)	**4.33E+08 MN m**	**4.33E+11 kN m**

mixed mode

circular y-rx	Ky-rx	**1.69E+05 MN**	**1.69E+08 kN**

mixed mode

circular x-ry	Kx-ry	**1.58E+05 MN**	**1.58E+08 kN**